PHY180 11-03

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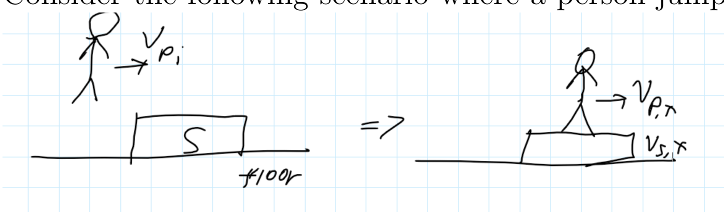
1 Person Jumping on a Slider

Consider the following scenario where a person jumps onto a moving slider:



Initial conditions:

$$m_p = m_s$$

$$\boldsymbol{\Delta} K = K_f - K_i = \frac{1}{2}(M_p + M_s)V_f^2 - \frac{1}{2}M_pV_i^2$$

$$= mv_f^2 - \frac{1}{2}mv_i^2$$

$$m(\frac{v_i}{2}^2) - \frac{1}{2}m_1v_1^2 = -\frac{1}{4}mv_i^2$$

3.

$$\boldsymbol{\Delta} E = 0$$

$$\boldsymbol{\Delta} E_{thermal} + \delta K = 0$$

$$\boldsymbol{\Delta} E_{thermal=F_{kinetic} \cdot d_{relative}}$$

$$\boldsymbol{\Delta} E_{thermal} = -\boldsymbol{\Delta} K + \frac{1}{4}mv_1^2$$

$$d_{relative} = \frac{V_i^2}{4\mu_k g}$$

2 Power as a product of vectors

Goal is to generalize from:

$$P = F_x v_x$$

$$\frac{dK}{dt} = \frac{1}{2} m \frac{d}{dt} (v^2)_x = m v_x a_x = F_x v_x = Power$$

Multiple dimensions:

$$K = \frac{1}{2} m v^2$$

$$P = \frac{1}{2} m \frac{d}{dt} |\overrightarrow{v}| = (2 v_x a_x + 2 v_y a_y + 2 v_z a_z)$$

$$m(a_x v_x + a_y v_y + a_z v_z)$$

$$= F_x v_x + f_y v_y + F_z v_z$$

$$= \overrightarrow{F} \cdot \overrightarrow{v}$$

Work done at an interval dt in 1d:

$$dW = F_x dx$$

$$dW = \overrightarrow{F} \cdot \delta \overrightarrow{r}$$

$$W = \int_{x_i}^{x_f} F_x(x) dx$$

Now with vectors, this is called a line integral:

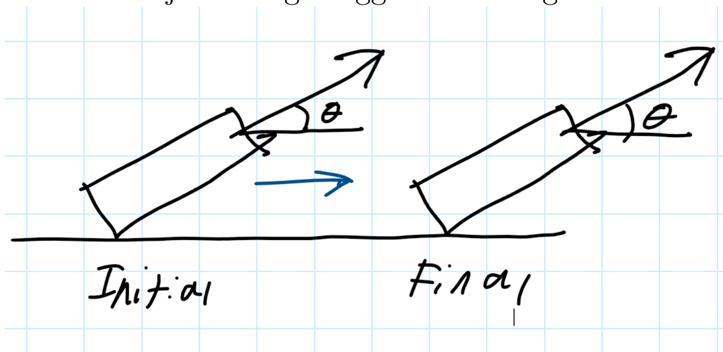
$$W = \int_{\overrightarrow{r_i}}^{\overrightarrow{r_f}} \overrightarrow{F}(\overrightarrow{r}) \cdot \delta \overrightarrow{r}$$

Special case of constant force: ...some algebra...

$$W = \overrightarrow{F} \cdot \boldsymbol{\Delta} \overrightarrow{r}$$

3 Work done by constant (2d) forces

Consider an object being dragged at an angle of θ

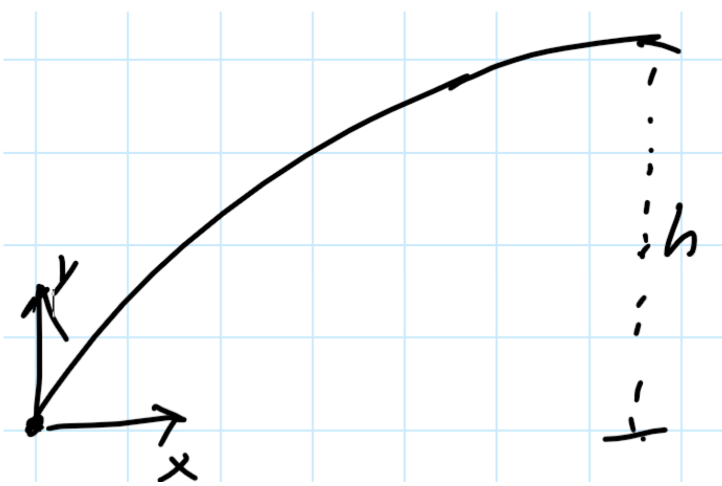


$$W = \overrightarrow{F} \cdot \boldsymbol{\Delta}\overrightarrow{r}$$

$$= \left|\overrightarrow{F}\right| \left|\overrightarrow{\boldsymbol{\Delta} r}\right| cos\theta$$

If $\theta = 0°$ no work is done! $(cos(90) = 0)$

4 Projectile Example



$$W = \overrightarrow{F} \cdot \boldsymbol{\Delta}\overrightarrow{r}$$

$$= mg(-y_{hat} \cdot (\boldsymbol{\Delta} x \boldsymbol{\Delta} y))$$
$$= -mg\boldsymbol{\Delta} y$$
$$= -mg\boldsymbol{\Delta} h$$

For a full trip up and down to same y position *no work is done.*

5 Force Created by Potential Energy

$$F_x = -\frac{dU}{dx}$$

Minus sign is very important. This equation can be extended to any direction such as y or z by looking at different derivatives

$$\overrightarrow{F} = -\frac{dU}{dx}x^{hat} - \frac{dU}{dy}y^{hat}$$

6 Gravity Near Earth

$$U(x, y) = mgy$$
$$\overrightarrow{F} = (0, -mg)$$

7 Spring on a Wall

$$U(x, y) = \frac{1}{2}kx^2$$
$$F(X) = -\frac{dU}{dx} = -kx$$
$$\overrightarrow{F} = (-kx, 0)$$

8 Gravity Far From Earth

$$U(r) = -\frac{GMm}{r}$$

$$F_r = -\frac{dU}{dr}$$

$$F_r^{gravity} = -\frac{GMm}{r^2}$$

9 Conservative Forces

Work done by a conservative force conserves U + k is independent of path taken.

In one dimension:

$$W = \int_{x_1}^{x_2} \overrightarrow{F} dx = \int_{x_1}^{x_2} -\frac{dU}{dx} dx$$

$$= -[U(x_f) - U(x_i)]$$

In two dimensions:

$$W = -[U(x_f, y_f) - U(x_i, y_i))]$$